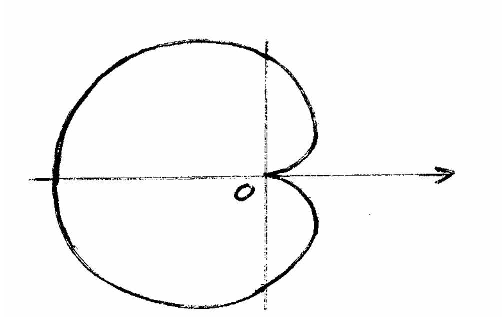
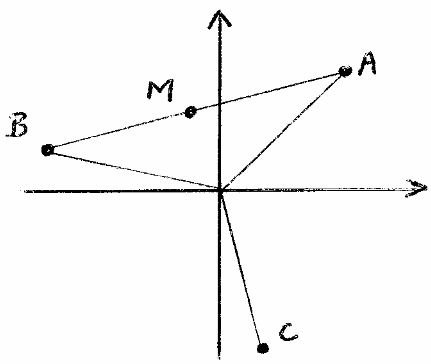
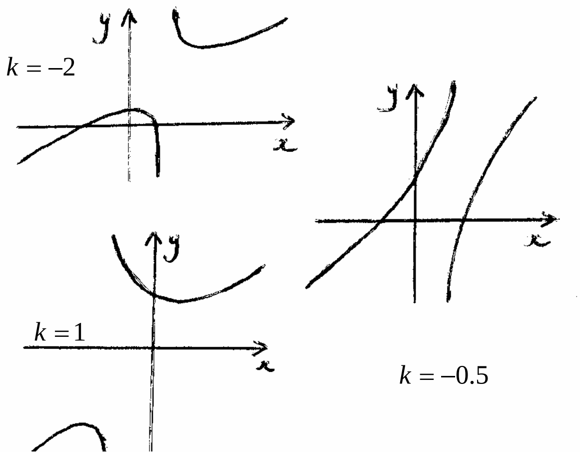
1(a)(i)		B2	2	Must include a sharp point at O and have infinite gradient at $\theta = \pi$ Give B1 for r increasing from zero for $0 < \theta < \pi$, or decreasing to zero for $-\pi < \theta < 0$
(ii)	Area is $\int \frac{1}{2} r^2 \, d\theta = \int_0^{\frac{1}{2}\pi} \frac{1}{2} a^2 (1 - \cos\theta)^2 \, d\theta$	M1 A1		For integral of $(1-\cos\theta)^2$ For a correct integral expression including limits *(may be implied by later work)*
	$= \frac{1}{2} a^2 \int_0^{\frac{1}{2}\pi} \left(1 - 2\cos\theta + \frac{1}{2}(1+\cos 2\theta) \right) d\theta$	B1		Using $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$
	$= \frac{1}{2} a^2 \left[\frac{3}{2}\theta - 2\sin\theta + \frac{1}{4}\sin 2\theta) \right]_0^{\frac{1}{2}\pi}$	B1B1 ft		Integrating $a + b\cos\theta$ and $k\cos 2\theta$
	$= \frac{1}{2} a^2 (\frac{3}{4}\pi - 2)$	B1	6	Accept $0.178a^2$
(b)	Put $x = 2\sin\theta$	M1		or $x = 2\cos\theta$
	Integral is $\int_0^{\frac{1}{6}\pi} \frac{1}{(4 - 4\sin^2\theta)^{\frac{3}{2}}} (2\cos\theta) \, d\theta$	A1		Limits not required
	$= \int_0^{\frac{1}{6}\pi} \frac{2\cos\theta}{8\cos^3\theta} d\theta = \int_0^{\frac{1}{6}\pi} \frac{1}{4}\sec^2\theta \, d\theta$			
	$= \left[\frac{1}{4}\tan\theta \right]_0^{\frac{1}{6}\pi}$	M1		For $\int \sec^2\theta \, d\theta = \tan\theta$
	$= \frac{1}{4} \times \frac{1}{\sqrt{3}} = \frac{1}{4\sqrt{3}}$	A1 ag	4	*SR* If $x = 2\tanh u$ is used M1 for $\frac{1}{4}\sinh(\frac{1}{2}\ln 3)$ A1 for $\frac{1}{8}(\sqrt{3} - \frac{1}{\sqrt{3}}) = \frac{1}{4\sqrt{3}}$ (max 2 / 4)
(c)(i)	$f'(x) = \frac{-2}{\sqrt{1-4x^2}}$	B2	2	Give B1 for any non-zero real multiple of this (or for $\frac{-2}{\sin y}$ etc)
(ii)	$f'(x) = -2(1-4x^2)^{-\frac{1}{2}}$	M1		Binomial expansion (3 terms, $n = -\frac{1}{2}$)
	$= -2(1 + 2x^2 + 6x^4 + ...)$	A1		Expansion of $(1-4x^2)^{-\frac{1}{2}}$ correct (accept unsimplified form)
	$f(x) = C - 2x - \frac{4}{3}x^3 - \frac{12}{5}x^5 + ...$	M1		Integrating series for $f'(x)$
	$f(0) = \frac{1}{2}\pi \Rightarrow C = \frac{1}{2}\pi$			
	$f(x) = \frac{1}{2}\pi - 2x - \frac{4}{3}x^3 - \frac{12}{5}x^5 + ...$	A1	4	Must obtain a non-zero x^5 term C not required

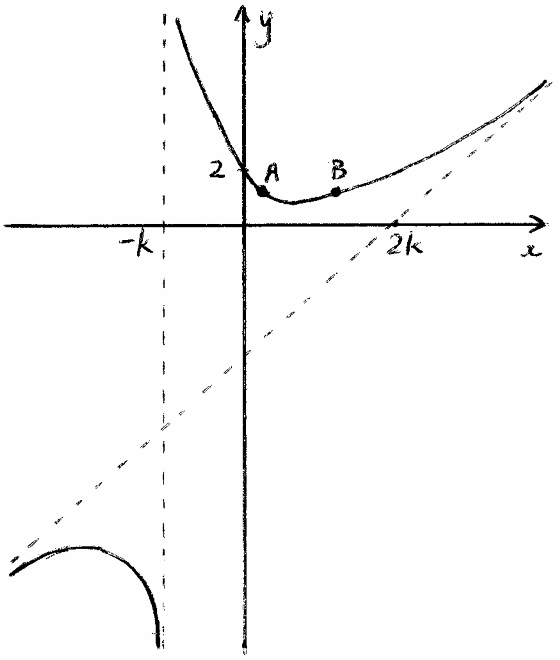
	OR by repeated differentiation			
	Finding $\mathrm{f}^{(5)}(x)$	M1		
	Evaluating $\mathrm{f}^{(5)}(0) \quad (=-288)$	M1		Must obtain a non-zero value
	$\mathrm{f}'(x) = -2 - 4x^2 - 12x^4 + \ldots$	A1 ft		ft from (c)(i) when B1 given
	$\mathrm{f}(x) = \frac{1}{2}\pi - 2x - \frac{4}{3}x^3 - \frac{12}{5}x^5 + \ldots$	A1		

2 (a)	$(\cos\theta + \mathrm{j}\sin\theta)^5$ $= c^5 + 5\mathrm{j}c^4s - 10c^3s^2 - 10\mathrm{j}c^2s^3 + 5cs^4 + \mathrm{j}s^5$ Equating imaginary parts $\sin 5\theta = 5c^4s - 10c^2s^3 + s^5$ $= 5(1-s^2)^2 s - 10(1-s^2)s^3 + s^5$ $= 5s - 10s^3 + 5s^5 - 10s^3 + 10s^5 + s^5$ $= 5\sin\theta - 20\sin^3\theta + 16\sin^5\theta$	M1 M1 A1 M1 A1 ag **5**	
(b)(i)	$\lvert -2+2\mathrm{j} \rvert = \sqrt{8}$, $\arg(-2+2\mathrm{j}) = \frac{3}{4}\pi$ $r = \sqrt{2}$ $\theta = \frac{1}{4}\pi$ $\theta = \frac{11}{12}\pi,\ -\frac{5}{12}\pi$	B1B1 B1 ft B1 ft M1 A1 **6**	Accept 2.8; 2.4, 135° (Implies B1 for $\sqrt{8}$) One correct (Implies B1 for $\frac{3}{4}\pi$) Adding or subtracting $\frac{2}{3}\pi$ Accept $\theta = \frac{1}{4}\pi + \frac{2}{3}k\pi$, $k = 0, 1, -1$
(ii)		B2 **2**	Give B1 for two of B, C, M in the correct quadrants Give B1 ft for all four points in the correct quadrants
(iii)	$\lvert w \rvert = \frac{1}{2}\sqrt{2}$ $\arg w = \frac{1}{2}(\frac{1}{4}\pi + \frac{11}{12}\pi) = \frac{7}{12}\pi$	B1 ft B1 **2**	Accept 0.71 Accept 1.8
(iv)	$\lvert w^6 \rvert = (\frac{1}{2}\sqrt{2})^6 = \frac{1}{8}$ $\arg(w^6) = 6 \times \frac{7}{12}\pi = \frac{7}{2}\pi$ $w^6 = \frac{1}{8}(\cos\frac{7}{2}\pi + \mathrm{j}\sin\frac{7}{2}\pi)$ $= -\frac{1}{8}\mathrm{j}$	M1 A1 ft A1 **3**	Obtaining either modulus or argument Both correct (ft) Allow from $\arg w = \frac{1}{4}\pi$ etc
			SR If B, C interchanged on diagram (ii) B1 (iii) B1 B1 for $-\frac{1}{12}\pi$ (iv) M1A1A1

3 (i)	$\det(\mathbf{M}-\lambda\mathbf{I}) = (3-\lambda)[(3-\lambda)(-4-\lambda)-4]$ $-5[5(-4-\lambda)+4]+2[-10-2(3-\lambda)]$ $=(3-\lambda)(-16+\lambda+\lambda^2)-5(-16-5\lambda)+2(-16+2\lambda)$ $=-48+19\lambda+2\lambda^2-\lambda^3+80+25\lambda-32+4\lambda$ $=48\lambda+2\lambda^2-\lambda^3$ Characteristic equation is $\lambda^3-2\lambda^2-48\lambda=0$	M1 A1 M1 A1 ag **4**	Obtaining $\det(\mathbf{M}-\lambda\mathbf{I})$ Any correct form Simplification
(ii)	$\lambda(\lambda-8)(\lambda+6)=0$ Other eigenvalues are $8,\ -6$ When $\lambda=8$, $3x+5y+2z=8x$ $(\ 5x+3y-2z=8y\ )$ $2x-2y-4z=8z$ $y=x$ and $z=0$; eigenvector is $\begin{pmatrix}1\\1\\0\end{pmatrix}$ When $\lambda=-6$, $3x+5y+2z=-6x$ $5x+3y-2z=-6y$ $y=-x,\ z=-2x$; eigenvector is $\begin{pmatrix}1\\-1\\-2\end{pmatrix}$	M1 A1 M1 M1 A1 M1 M1 A1 **8**	Solving to obtain a non-zero value Two independent equations Obtaining a non-zero eigenvector *($-5x+5y+2z=8x$ etc can earn M0M1)* Two independent equations Obtaining a non-zero eigenvector
(iii)	$\mathbf{P}=\begin{pmatrix}1&1&1\\-1&1&-1\\1&0&-2\end{pmatrix}$ $\mathbf{D}=\begin{pmatrix}0&0&0\\0&8&0\\0&0&-6\end{pmatrix}^2$ $=\begin{pmatrix}0&0&0\\0&64&0\\0&0&36\end{pmatrix}$	B1 ft M1 A1 **3**	B0 if **P** is clearly singular Order must be consistent with **P** when B1 has been earned
(iv)	$\mathbf{M}^3-2\mathbf{M}^2-48\mathbf{M}=\mathbf{0}$ $\mathbf{M}^3=2\mathbf{M}^2+48\mathbf{M}$ $\mathbf{M}^4=2\mathbf{M}^3+48\mathbf{M}^2$ $=2(2\mathbf{M}^2+48\mathbf{M})+48\mathbf{M}^2$ $=52\mathbf{M}^2+96\mathbf{M}$	M1 M1 A1 **3**	

4 (a)	$\int_0^1 \frac{1}{\sqrt{9x^2+16}}\,dx = \left[\frac{1}{3}\operatorname{arsinh}\frac{3x}{4}\right]_0^1$ $= \frac{1}{3}\operatorname{arsinh}\frac{3}{4}$ $= \frac{1}{3}\ln(\frac{3}{4}+\sqrt{\frac{9}{16}+1})$ $= \frac{1}{3}\ln 2$	M1 A1 A1 M1 A1 **5**	For arsinh or for any sinh substitution For $\frac{3}{4}x$ or for $3x = 4\sinh u$ For $\frac{1}{3}$ or for $\int \frac{1}{3}du$
	OR M2 $\left[\frac{1}{3}\ln(3x+\sqrt{9x^2+16})\right]_0^1$ A1A1 $= \frac{1}{3}\ln 8 - \frac{1}{3}\ln 4$ $= \frac{1}{3}\ln 2$ A1		For $\ln(kx+\sqrt{k^2x^2+...})$ [Give M1 for $\ln(ax+\sqrt{bx^2+...})$] or $\frac{1}{3}\ln(x+\sqrt{x^2+\frac{16}{9}})$
(b)(i)	$2\sinh x\cosh x = 2\times\frac{1}{2}(e^x-e^{-x})\frac{1}{2}(e^x+e^{-x})$ $= \frac{1}{2}(e^{2x}-e^{-2x})$ $= \sinh 2x$	M1 A1 **2**	$(e^x-e^{-x})(e^x+e^{-x}) = (e^{2x}-e^{-2x})$ For completion
(ii)	$\frac{dy}{dx} = 20\sinh x - 6\sinh 2x$ For stationary points, $20\sinh x - 12\sinh x\cosh x = 0$ $4\sinh x(5-3\cosh x) = 0$ $\sinh x = 0$ or $\cosh x = \frac{5}{3}$ $x=0,\quad y=17$ $x = (\pm)\ln(\frac{5}{3}+\sqrt{\frac{25}{9}-1}) = \ln 3$ $y = 10\left(3+\frac{1}{3}\right)-\frac{3}{2}\left(9+\frac{1}{9}\right) = \frac{59}{3}$ $x = -\ln 3,\quad y = \frac{59}{3}$	B1B1 M1 A1 A1 ag A1 ag B1 **7**	When exponential form used, give B1 for any 2 terms correctly differentiated Solving $\frac{dy}{dx}=0$ to obtain a value of $\sinh x$, $\cosh x$ or e^x (or $x=0$ stated) Correctly obtained Correctly obtained *The last A1A1 ag can be replaced by B1B1 ag for a full verification*
(iii)	$\left[20\sinh x - \frac{3}{2}\sinh 2x\right]_{-\ln 3}^{\ln 3}$ $= \left\{10\left(3-\frac{1}{3}\right)-\frac{3}{4}\left(9-\frac{1}{9}\right)\right\}\times 2$ $= \left(\frac{80}{3}-\frac{20}{3}\right)\times 2 = 40$	B1B1 M1 A1 ag **4**	When exponential form used, give B1 for any 2 terms correctly integrated Exact evaluation of $\sinh(\ln 3)$ and $\sinh(2\ln 3)$

5 (i)		B1 B1 B1 B1 B1 B1 **6**	Maximum on LH branch and minimum on RH branch Crossing axes correctly Two branches with positive gradient Crossing axes correctly Maximum on LH branch and minimum on RH branch Crossing positive *y*-axis and minimum in first quadrant
(ii)	$y=\dfrac{(x+k)(x-2k)+2k^2+2k}{x+k}$ $=x-2k+\dfrac{2k(k+1)}{x+k}$ Straight line when $2k(k+1)=0$ $k=0,\quad k=-1$	M1 A1 (ag) B1B1 **4**	Working in either direction For completion
(iii)(*A*)	Hyperbola	B1 **1**	
(*B*)	$x=-k$ $y=x-2k$	B1 B1 **2**	

(iv)		B1 B1 B1 B1 B1 **5**	Asymptotes correctly drawn Curve approaching asymptotes correctly (both branches) Intercept 2 on *y*-axis, and not crossing the *x*-axis Points A and B marked, with minimum point between them Points A and B at the same height $(y = 1)$